

# Soft Supersymmetry Breaking Parameters and Minimal SO(10) Unification\*

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## Abstract

The minimal supersymmetric SO(10) model, in which not only the gauge but also the third generation fermion Yukawa couplings are unified, provides a simple and highly predictive theoretical scenario for the understanding of the origin of the low energy gauge interactions and fermion masses. In the framework of the Minimal Supersymmetric Standard Model with universal soft supersymmetry breaking parameters at the grand unification scale, large values of the universal gaugino mass  $M_{1/2} \geq 300$  GeV are needed in order to induce a proper breakdown of the electroweak symmetry. In addition, in order to obtain acceptable experimental values for both the pole bottom mass and the  $b \rightarrow s\gamma$  decay rate, even larger values of the gaugino masses are required. The model is strongly constrained by theoretical and phenomenological requirements and a heavy top quark, with mass  $M_t \geq 170$  GeV, is hard to accomodate within this scheme. We show, however, that it is sufficient to relax the condition of universality of the scalar soft supersymmetry breaking parameters at the grand unification scale to be able to accomodate a top quark mass  $M_t \simeq 180$  GeV. Still, the requirement of a heavy top quark demands a very heavy squark spectrum, unless specific relations between the soft supersymmetry breaking parameters are fulfilled.

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# 1 Introduction

The Standard Model provides a very good understanding of the strong and electroweak interactions and, so far, it has withstood all the experimental onslaughts. Yet, it leaves a host of open questions, which require, to be answered, the presence of an underlying structure with a larger symmetry content than the one sufficient to describe the physics at present accelerator energies. In particular, an explanation to the origin of forces and fermion masses, as well as the hierarchy between the Planck and the weak scale is still lacking. Supersymmetric theories have the potential of dealing with these problems and, at the same time, of remaining compatible with the low energy data. This is why, in spite of the lack of experimental evidence for supersymmetry, most theories beyond the Standard Model include this symmetry at some stage. In the simplest supersymmetric theories, gauge invariant, soft supersymmetry breaking terms are present, which yield masses to the unobserved supersymmetric particles, while the standard model fermion and boson fields acquire masses through the usual Higgs mechanism [1]. In addition, supersymmetry ensures the stability of the hierarchy between the weak and the Planck scales. Moreover, in the minimal supersymmetric models the weak scale and the scale of supersymmetry breaking are interrelated, and a natural explanation of the scale of electroweak symmetry breaking may only be achieved if the supersymmetric particle masses are smaller than, or of the order of, 1 TeV.

It has recently been realized that the weak and strong gauge couplings determined by the most recent measurements at the LEP experiments are consistent with the unification of gauge couplings within the minimal supersymmetric standard model [2]. The presence of gauge coupling unification has strongly revived the interest in softly broken supersymmetric theories, in particular, in the Minimal Supersymmetric extension of the Standard Model. Much work has been done in understanding the influence of threshold corrections at low and high energy scales and the impact of the precise supersymmetric spectrum on the predictions for the strong gauge coupling [3]. In particular, it has been shown that the low energy threshold corrections are strongly dependent on the Higgsino and gaugino masses, and weakly dependent on the squark and sfermion masses [4]. Moreover, the relevance of the experimental correlation between the top quark mass and the weak mixing angle in the obtention of the strong gauge coupling predictions has been emphasized. As we shall discuss below, due to this correlation, a heavy top quark, with mass  $M_t > 150$  (170) GeV, is associated with values of the strong gauge coupling  $\alpha_3(M_Z) \geq 0.114$  (0.117). The lower bound on  $\alpha_3(M_Z)$  increases for larger values of the

top quark mass.

Furthermore, the question of fermion masses may also find a natural explanation within minimal supersymmetric Grand Unified Theories (see, for example, Refs. [5],[6]). In fact, minimal supersymmetric GUTs give a natural explanation for the heaviness of the top quark: The condition of bottom–tau Yukawa coupling unification [7],[8] requires large values of the top quark Yukawa coupling,  $h_t$ , at the grand unification scale,  $M_{GUT}$ , in order to contravene the strong gauge coupling effects on the running of the bottom Yukawa coupling [3]–[5],[9]. Moreover, if the top quark Yukawa coupling acquires large values at the grand unification scale,  $Y_t = h_t^2/4\pi \geq 0.1$ , its low energy value is completely determined by the infrared fixed point structure of the theory [10]–[12]. Indeed, for small and moderate values of  $\tan\beta$  –the ratio of vacuum expectation values of the Higgs fields– for which the effects of the bottom quark Yukawa coupling in the running of  $h_t$  may be safely neglected, the infrared quasi-fixed point value of the running top quark mass in the  $\overline{MS}$  scheme is approximately given by

$$m_t(M_t)^{IR} \simeq 196 \text{ GeV} [1 + 2 (\alpha_3(M_Z) - 0.12)] \sin\beta \quad (1)$$

where the pole mass is related to the running mass by [13]

$$M_t \simeq m_t(M_t) \left[ 1 + 4\alpha_3(M_t)/3\pi + 11(\alpha_3(M_t)/\pi)^2 \right]. \quad (2)$$

A careful analysis shows that, for small and moderate values of  $\tan\beta$  the conditions of gauge and bottom–tau Yukawa coupling unification lead to a top quark mass which, for the currently acceptable values for the bottom mass and the electroweak parameters, is within 10% of its infrared fixed point value [14],[15].

In general, the condition of bottom–tau Yukawa coupling unification determines the value of the top quark Yukawa coupling and, in the small and moderate  $\tan\beta$  regime, implies a strong attraction of the top quark mass to its infrared fixed point. Observe that the infrared fixed point solution does not provide a direct prediction for the top quark mass, but only a strong correlation between  $M_t$  and  $\tan\beta$ . A prediction for both the top quark mass and  $\tan\beta$  may only be obtained in theories with a richer symmetry structure than the one provided by the minimal supersymmetric SU(5) theory. In this respect, the minimal SO(10) model, where all Yukawa couplings proceed from a common coupling of the 16 representation of matter fields with a 10 representation of Higgs fields, provides a natural extension of the SU(5) scenario [6],[16]–[20]. Since the top and bottom Yukawa couplings are of the same order, the hierarchy of top and bottom masses is due to a large value of the ratio of vacuum expectation values:

$$\tan\beta \simeq \frac{m_t(m_t)}{m_b(m_t)} \simeq \mathcal{O}(50). \quad (3)$$

The condition of bottom–tau Yukawa unification is implicit within this scheme, therefore, the infrared fixed point attraction is potentially present and the top quark mass tends to get larger values. However, since the bottom and the top Yukawa couplings are of the same order, the bottom Yukawa effect is sufficiently strong by itself to partially contravene the strong gauge coupling effects on its renormalization group running. Then, the top quark Yukawa coupling at the grand unification scale tends to be smaller than for moderate values of  $\tan\beta$  and the infrared fixed point attraction becomes weaker. The top quark mass prediction becomes much more sensitive to the actual value of the bottom mass and the strong gauge coupling. Hence, the strong predictivity expected from the combination of large values of  $\tan\beta$  and the infrared fixed point attraction is not actually realized within this scheme.

Moreover, for the large values of  $\tan\beta$  implied by the above condition, Eq. (3), large corrections to the running bottom mass induced by the supersymmetry breaking sector of the theory are present [6],[21]–[23]. These corrections, which may be as large as 50% of the bottom mass value, make the top quark mass predictions highly dependent on the nature of the supersymmetry breaking sector of the theory. For instance, if the soft supersymmetry breaking parameters are such that these corrections are negligible, larger values of the top quark mass  $M_t \geq 170$  GeV are preferred. In the case of universal soft supersymmetry breaking parameters at the grand unification scale, these corrections are, instead, large, and lower values of the top quark mass,  $M_t \leq 170$  GeV, are preferred [26]. Hence, no prediction for the top quark mass may be obtained unless a specific framework for the breakdown of supersymmetry is given.

In this talk, we first concentrate on the simplest supersymmetry breaking scenario, with universal soft supersymmetry breaking parameters at the grand unification scale, and we describe the phenomenological and theoretical constraints arising in this model (for a detailed discussion of similar issues at low values of  $\tan\beta$ , we refer the reader to [24] and [25]). We shall show that strong correlations between the different sparticle masses appear within this scheme. Furthermore, a lower bound on the squark and gaugino masses is obtained from the requirement of a proper  $SU(2)_L \times U(1)_Y$  breakdown. We shall also discuss the corrections to the bottom mass, its correlation with the supersymmetric contributions to the  $b \rightarrow s\gamma$  decay rate and its implication for the top quark mass predictions. Finally, we shall briefly describe the implications of relaxing the condition of universality

of the soft supersymmetry breaking parameters, both in the Higgs and the sfermion sector of the theory.

## 2 Gauge Coupling Unification

In order to analyse the condition of gauge coupling unification, the experimental correlation between the top quark pole mass and the weak mixing angle should be considered. Indeed, taking as input values the Fermi constant, the value of the  $Z$ -boson mass  $M_Z$ , and the value of  $\alpha_{em}(M_Z)$ , in the modified  $\bar{M}\bar{S}$  scheme a correlation between the top quark mass and  $\sin^2 \theta_W(M_Z)$  is induced through the top quark mass dependent radiative corrections to the weak mixing angle [3],

$$\sin^2 \theta_W(M_Z) = 0.2324 - 10^{-7} \left( M_t^2 - (138 \text{ GeV})^2 \right) \text{GeV}^{-2} \pm 0.003. \quad (4)$$

In addition, in order to fully understand the implications of gauge coupling unification, a few words about the supersymmetric threshold corrections to the gauge couplings should be said. For a given supersymmetric spectrum, and a fixed value of the weak mixing angle, the value of  $\alpha_3(M_Z)$ , determined by the gauge coupling unification condition, is given by

$$\begin{aligned} \frac{1}{\alpha_3(M_Z)} &= \frac{(b_1 - b_3)}{(b_1 - b_2)} \left[ \frac{1}{\alpha_2(M_Z)} + \gamma_2 + \Delta_2 \right] - \frac{(b_2 - b_3)}{(b_1 - b_2)} \left[ \frac{1}{\alpha_1(M_Z)} + \gamma_1 + \Delta_1 \right] \\ &- \gamma_3 - \Delta_3 + \Delta^{Sthr} \left( \frac{1}{\alpha_3(M_Z)} \right), \end{aligned} \quad (5)$$

where

$$\Delta^{Sthr} \left( \frac{1}{\alpha_3(M_Z)} \right) = \frac{19}{28\pi} \ln \left( \frac{T_{SUSY}}{M_Z} \right) \quad (6)$$

is the contribution to  $1/\alpha_3(M_Z)$  due to the inclusion of the supersymmetric threshold corrections at the one-loop level,  $\gamma_i$  includes the two-loop corrections to the value of  $1/\alpha_i(M_Z)$ ,  $\Delta_i$  are correction constants that allow a transformation of the gauge couplings from the minimal  $\bar{M}\bar{S}$  scheme to the dimensional reduction scheme  $\bar{D}\bar{R}$ , more appropriate for supersymmetric theories, and  $b_i$  are the supersymmetric beta function coefficients associated to the gauge coupling  $\alpha_i$ . As becomes clear from Eq. (6), the effective threshold scale  $T_{SUSY}$  gives a parametrization of the size of the supersymmetric threshold corrections to the gauge couplings and it would coincide with the overall mass scale  $M_{SUSY}$  only if all supersymmetric particles were degenerate in mass.

In order to study the dependence of  $T_{SUSY}$  on the different sparticle masses, we define  $m_{\tilde{q}}$ ,  $m_{\tilde{g}}$ ,  $m_{\tilde{l}}$ ,  $m_{\tilde{W}}$ ,  $m_{\tilde{H}}$  and  $m_H$  as the characteristic masses of the squarks, gluinos, sleptons, electroweak gauginos, Higgsinos and the heavy Higgs doublet, respectively. Assuming different values for all these mass scales, we derive an expression for the effective supersymmetric threshold  $T_{SUSY}$ , which is given by [4]

$$T_{SUSY} = m_{\tilde{H}} \left( \frac{m_{\tilde{W}}}{m_{\tilde{g}}} \right)^{28/19} \left[ \left( \frac{m_{\tilde{l}}}{m_{\tilde{q}}} \right)^{3/19} \left( \frac{m_H}{m_{\tilde{H}}} \right)^{3/19} \left( \frac{m_{\tilde{W}}}{m_{\tilde{H}}} \right)^{4/19} \right]. \quad (7)$$

The above relation holds whenever all the particles involved have masses  $m_\eta > M_Z$ . If, instead, any of the sparticles or the heavy Higgs boson has a mass  $m_\eta < M_Z$ , it should be replaced by  $M_Z$  for the purpose of computing the supersymmetric threshold corrections to  $1/\alpha_3(M_Z)$ . From Eq. (7), it follows that  $T_{SUSY}$  has only a slight dependence on the squark, slepton and heavy Higgs masses and a very strong dependence on the overall Higgsino mass, as well as on the ratio of masses of the gauginos associated with the electroweak and strong interactions. In Table 1, we show the predictions for the strong gauge coupling, for different values of  $\sin^2 \theta_W(M_Z)$  and the supersymmetric threshold scale, together with the approximate value of the top quark pole mass associated with each value of  $\sin^2 \theta_W(M_Z)$ .

$M_t$ [GeV]	$\sin^2 \theta_W(M_Z)$	$\alpha_3(M_Z)$ for $T_{SUSY} = 1$ TeV	$\alpha_3(M_Z)$ for $T_{SUSY} = 100$ GeV
140	0.2324	0.116	0.123
170	0.2315	0.119	0.127
195	0.2305	0.123	0.131

**Table 1.** Dependence of  $\alpha_3(M_Z)$  on  $\sin^2 \theta_W(M_Z)$  and  $T_{SUSY}$ , in the framework of gauge coupling unification.

The above values of  $\alpha_3(M_Z)$  are obtained from our two-loop renormalization group analysis, and they depend slightly on the top quark Yukawa coupling, and hence on  $\tan \beta$ . In general, for values of the top quark mass  $M_t \geq 140$  GeV, the variation induced through the top quark Yukawa coupling contribution is at most 1% of the values given above (with lower values of  $\alpha_3(M_Z)$  obtained for larger values of the top quark Yukawa coupling). Observe that in models with universal gaugino masses at the grand unification scale and for large values of the supersymmetric mass parameters, for which the mixing in the neutralino and chargino sectors may be neglected,  $T_{SUSY} \simeq \mu/6$ , where  $\mu$  is the

supersymmetric mass parameter appearing in the superpotential. Hence, a value of  $T_{SUSY}$  of the order of 1 TeV implies that the supersymmetric spectrum contains sparticles with masses far above the TeV scale. If all supersymmetric masses are below or of the order of 1 TeV, the effective supersymmetric threshold scale is below or of the order of the weak scale.

The largest uncertainties associated with the unification scheme come from the threshold corrections at the grand unification scale. We shall not discuss them here (for a detailed discussion, see, for example, Ref. [3]), but we shall assume moderate corrections, of the order of those coming from the supersymmetric spectrum. Thus, for a supersymmetric spectrum with characteristic masses of the order of or below 1 TeV, the condition of gauge coupling unification, together with the experimental correlation between  $\sin^2 \theta_W(M_Z)$  and  $M_t$ , Eq. (4), imply the following correlation between  $\alpha_3(M_Z)$  and the top quark mass,

$$M_t^2 \simeq (138 \text{ GeV})^2 + 0.25 \times 10^7 \text{ GeV}^2 (\alpha_3(M_Z) - 0.123 \pm 0.01). \quad (8)$$

It is instructive to compare Eq. (8) with the results presented in table 1. As we discussed in section 1, a lower bound for  $\alpha_3(M_Z)$  as a function of the top quark mass is derived. From Eq. (8) it follows that, for a top quark mass  $M_t \geq 150$  (170) GeV, the values of the strong gauge coupling are  $\alpha_3(M_Z) \geq 0.114$  (0.117).

### 3 Yukawa Coupling Unification

The general features of Yukawa coupling unification in the Minimal Supersymmetric Standard Model are discussed in section 1. In the following, we shall concentrate on the properties of the minimal supersymmetric SO(10) model, for which not only the bottom and the tau, but also the top quark Yukawa coupling unify at the GUT scale. As we discussed in section 1, large values of  $\tan \beta$  ( $\simeq \mathcal{O}(50)$ ), are predicted within this scheme.

The fact that in the minimal supersymmetric SO(10) model, the value of  $\tan \beta$  is approximately equal to the ratio of the top and bottom quark masses at the top mass scale comes from the approximate equality of the top and bottom Yukawa couplings at low energies. Such an approximate equality is implied by their unification condition, and the presumption that the bottom and top quarks acquire masses, each of them, only through one of the two Higgs doublet vacuum expectation values:  $m_t(m_t) = h_t(m_t)v_2$ ,  $m_b(m_t) = h_b(m_t)v_1$ , where  $v_i$  is the vacuum expectation value of the Higgs  $H_i$ . This property holds in the supersymmetric limit and, within a good approximation, for small

and moderate values of  $\tan\beta$  when supersymmetry is softly broken. In general, however, a coupling of the bottom (top) quark to the Higgs  $H_2$  ( $H_1$ ) may be generated at the one-loop level. Although these couplings are small compared to  $h_b$  ( $h_t$ ) (typically lower than 1% of  $h_b$ ), for large values of  $\tan\beta$  –which implies  $v_2 \gg v_1$ – this may induce important corrections to the bottom mass [21]–[23], [26]:

$$m_b = h_b(v_1 + K_1 v_2) \equiv \tilde{m}_b \left(1 + \frac{\Delta m_b}{\tilde{m}_b}\right), \quad (9)$$

where  $K_1$  is the coefficient of the one-loop corrections to the bottom mass,  $\Delta m_b/\tilde{m}_b = K_1 \tan\beta$ , and  $\tilde{m}_b$  would be the value of the running bottom mass if the supersymmetric one-loop corrections were negligible. Recalling that, at the two-loop level, the physical and running bottom masses are related by

$$M_b = m_b(M_b) \left(1 + \frac{4}{3\pi} \alpha_3(M_b) + 12.4 \left(\frac{\alpha_3(M_b)}{\pi}\right)^2\right), \quad (10)$$

due to the low energy renormalization group running of the bottom quark mass the following property is fulfilled,

$$\frac{\Delta M_b}{\tilde{M}_b} \simeq \frac{\Delta m_b(M_b)}{\tilde{m}_b(M_b)} \simeq \frac{\Delta m_b(M_Z)}{\tilde{m}_b(M_Z)}. \quad (11)$$

In the above,  $\tilde{M}_b$  would be the value of the physical bottom mass if no supersymmetric corrections were present, while  $\Delta M_b \simeq M_b - \tilde{M}_b$ .

The above property, Eq. (11), is relevant for the understanding of the top quark mass predictions coming from the unification of couplings. In Fig. 1, we present the predictions for the top quark mass as a function of  $\alpha_3(M_Z)$  for different values of the *uncorrected* bottom quark mass  $\tilde{M}_b$ , as well as for different values of  $\tan\beta$  [26]. From this figure we observe that the top quark mass predictions are strongly dependent on the exact value of the strong gauge coupling and the *uncorrected* bottom mass  $\tilde{M}_b$ . Notice that, if we restrict ourselves to the region of  $\alpha_3(M_Z)$  preferred by gauge coupling unification, Eq. (8) (to the right of the dotted line), for  $M_b \leq 5$  GeV, the top quark mass is pushed towards large values. In fact, if the bottom mass corrections were small,  $\tilde{M}_b \simeq M_b$ , for experimentally acceptable values of the bottom quark pole mass,  $M_b = 4.9 \pm 0.3$  GeV [27], the top quark mass would be  $M_t \geq 165$  GeV. However, as we shall show in section



5, large bottom quark mass corrections may significantly change this prediction.

Fig. 1. Top quark mass predictions as a function of the strong gauge coupling  $\alpha_3(M_Z)$ , with unification of the three Yukawa couplings of the third generation. Starting from above, the solid lines represent values of  $\tilde{M}_b$  equal to 4.6, 4.9, 5.2, 5.5 and 5.8 GeV. Analogously, the dashed lines represent values of  $\tan\beta$  equal to 60, 55, 50, 45 and 40. The long-dashed line represents the top quark mass fixed point value and the dotted line shows the region preferred by the gauge coupling unification condition as explained in the text.

For a clear interpretation of Fig. 1 it is important that, for large values of  $\tan\beta$ , the fixed point prediction, Eq. (1), is modified due to the non-negligible bottom quark Yukawa coupling effect in the running of the top quark Yukawa coupling [4],[26]. Indeed, for  $h_b \simeq h_t$  a more appropriate expression than Eq. (1) is

$$m_t(M_t)^{IR} \simeq 182 \text{ GeV} [1 + 2 (\alpha_3(M_Z) - 0.12)] \quad (12)$$

which, recalling the relation between the running and the pole masses, Eq. (2), describes within a good approximation the upper bound on the top quark mass (long-dashed line) shown in Fig. 1.

## 4 Supersymmetry and Electroweak Symmetry Breaking

As we mentioned above, in the large  $\tan\beta$  regime, the top quark mass predictions depend strongly on the nature of the soft supersymmetry breaking terms arising at low energies. In principle, the Minimal Supersymmetric Standard Model yields a multiplication of free parameters, which are only constrained by the requirement of avoiding a conflict with present experimental data. One of the strongest requirements is the absence of flavour changing neutral currents, which is fulfilled if the squarks of the first two generations are approximately degenerate in mass. It has been realized long ago that such is naturally the case if all soft supersymmetry breaking squark and gaugino mass parameters are universal at the grand unification scale. This supersymmetry breaking scheme appears naturally in minimal supergravity models, in which not only the squarks but also the Higgs fields acquire common soft supersymmetry breaking terms at high energies, and all the supersymmetry breaking terms at low energies may be given as a function of only four parameters: The universal scalar mass  $m_0$ , the universal gaugino mass  $M_{1/2}$ , and the universal trilinear and bilinear couplings  $A_0$  and  $B_0$ , which are associated with contributions to the high energy effective potential proportional to the trilinear and bilinear terms of the superpotential.

We shall first concentrate on the minimal supergravity SO(10) model. In section 6 we shall briefly discuss the situation where the condition of universality of the soft supersymmetry breaking parameters is relaxed. In general, the superpotential reads

$$P = \mu\epsilon_{ab}H_1^aH_2^b + h_t\epsilon_{ab}Q^aUH_2^b + h_\tau\epsilon_{ab}L^aEH_1^b + h_b\epsilon_{ab}Q^aDH_1^b, \quad (13)$$

where  $Q^T = (T_L, B_L)$  and  $L^T = (\nu_L^T, \tau_L)$  are the top-bottom and lepton left-handed doublets,  $U = T_L^c$ ,  $D = B_L^c$  and  $E = \tau_L^c$  are the right-handed top-squark, bottom-squark and stau, respectively, and  $\epsilon_{ab}$  is the antisymmetric tensor with  $\epsilon_{12} = -1$ . The scalar potential is given by

$$\begin{aligned} V = & m_1^2H_1^\dagger H_1 + m_2^2H_2^\dagger H_2 + m_Q^2Q^\dagger Q + m_U^2U^*U + m_D^2D^*D + m_L^2L^\dagger L + m_E^2E^*E \\ & + \epsilon_{ab}(m_3^2H_1^aH_2^b + A_th_tQ^aUH_2^b + A_bh_bQ^aDH_1^b + \text{h.c.}) + \text{q.t.} + \dots, \end{aligned} \quad (14)$$

where the Higgs field mass terms contain a part coming from the superpotential and another coming from the soft supersymmetry breaking parameters:  $m_i^2 = \mu^2 + m_{H_i}^2$ , with  $i = 1, 2$  and  $m_3^2 = B\mu$ . The dots stand for scalar trilinear  $F$ -terms and q.t. characterizes

the quartic terms in the scalar fields. The soft supersymmetry breaking parameters  $m_{H_i}^2$  have the following renormalization group evolution,

$$\begin{aligned} 4\pi \frac{dm_{H_1}^2}{dt} &= 3\alpha_2 M_2^2 + \alpha_1 M_1^2 - 3Y_b M_{D_{eff}}^2 - Y_\tau M_{E_{eff}}^2, \\ 4\pi \frac{dm_{H_2}^2}{dt} &= 3\alpha_2 M_2^2 + \alpha_1 M_1^2 - 3Y_t M_{U_{eff}}^2, \end{aligned} \quad (15)$$

where  $t = \ln[(M_{GUT}/Q)^2]$ ,  $Y_i = h_i^2/(4\pi)$ ,  $M_{D_{eff}}^2 = m_Q^2 + m_D^2 + m_{H_1}^2 + A_b^2$ ,  $M_{U_{eff}}^2 = m_Q^2 + m_U^2 + m_{H_2}^2 + A_t^2$  and  $M_{E_{eff}}^2 = m_L^2 + m_E^2 + m_{H_1}^2 + A_\tau^2$ .

The rest of the renormalization group equations may be found in the literature [28]. Let us just remark that, in the minimal supergravity scheme,  $M_{D_{eff}}$  and  $M_{U_{eff}}$  present very similar renormalization group evolutions when  $h_t \simeq h_b$ . Indeed, for bottom–top Yukawa coupling unification they only differ by the different hypercharge quantum numbers of the right bottom and top quarks, the slightly different running of the bottom and top Yukawa couplings, and the small tau Yukawa coupling effects (recall that the tau Yukawa coupling is renormalized to lower values than the bottom and top ones, due to the absence of strong gauge coupling effects in its one-loop renormalization group evolution).

Considering the renormalization group evolution of the mass parameters  $m_{H_1}^2$  and  $m_{H_2}^2$ , for  $M_{1/2}^2 \gg m_0^2$ , values of  $m_{H_1}^2 > m_{H_2}^2$  are obtained, mainly due to the difference between the hypercharge quantum numbers of the right bottom and top quarks and their supersymmetric partners. For  $m_0^2 \gg M_{1/2}^2$ , instead, the inverse hierarchy,  $m_{H_2}^2 > m_{H_1}^2$ , is obtained, mainly due to the  $\tau$ -Yukawa coupling effects. In general, considering the bottom–top Yukawa coupling unification condition and performing a complete numerical analysis it follows that [26]

$$m_{H_1}^2 - m_{H_2}^2 \simeq \alpha M_{1/2}^2 + \beta m_0^2, \quad (16)$$

with  $\alpha \simeq -\beta \simeq 0.1 - 0.2$ , depending on the proximity of the top quark Yukawa coupling to its infrared fixed point value  $h_f$  (the above range is obtained for  $Y_t/Y_f = h_t^2/h_f^2 = 0.6 - 0.95$ , respectively).

## 4.1 Radiative Electroweak Symmetry Breaking

In order to induce a proper breakdown of the electroweak symmetry, the following conditions need to be fulfilled,

$$\sin 2\beta = \frac{2m_3^2}{m_A^2} \quad (17)$$

and

$$\tan^2 \beta = \frac{m_1^2 + M_Z^2/2 + \text{r.c.}}{m_2^2 + M_Z^2/2 + \text{r.c.}}, \quad (18)$$

where  $m_A^2 \simeq m_1^2 + m_2^2 + \text{r.c.}$  is the mass of the CP-odd Higgs eigenstate and r.c. symbolizes one-loop radiative correction contributions, which depend logarithmically on the supersymmetry breaking scale, and become of the order of  $M_Z^2$  for a characteristic supersymmetric scale of the order of 1 TeV. We shall ignore these corrections in the following, since they are unessential for the qualitative understanding of the phenomena under discussion. They are included, however, in the numerical analysis.

In general, independently of the supersymmetry breaking mechanism, since  $\tan \beta$  is very large, in order to avoid extremely large values of  $m_1^2$  (or  $m_A^2$ ), the mass parameters  $m_2^2$  and  $m_3^2$  should fulfil the following properties

$$m_2^2 \simeq -\frac{M_Z^2}{2}, \quad m_3^2 \ll M_Z^2. \quad (19)$$

As was explained in Ref. [22], the second of these conditions can be obtained by assuming that there is a softly broken symmetry implying the smallness of the parameters  $B$  and/or  $\mu$ . The first condition is just a reflection of the fact that the vacuum expectation value of the Higgs  $H_2$  is the one that determines the electroweak vector boson masses. The fact that the CP-odd Higgs mass squared should be positive, together with Eq. (19), implies that

$$m_1^2 - m_2^2 > M_Z^2. \quad (20)$$

The above described properties are general in the sense that they do not depend on the supersymmetry breaking mechanism. If we consider the running of the Higgs mass parameters in the minimal supergravity model with bottom-top Yukawa unification, Eq. (16), strong implications follow from Eqs. (19) and (20),

$$M_{1/2} > m_0, \quad M_{1/2} \geq \frac{M_Z}{\sqrt{\alpha}}. \quad (21)$$

Analysing the RG evolution of the soft supersymmetry breaking parameters, Eq. (15), and taking into account the constraints of Eq. (21), an approximate solution for the Higgs mass parameter  $m_2^2$  is obtained,

$$m_2^2 \simeq \mu^2 + M_{1/2}^2 \left( 0.5 - C_1 \frac{Y}{Y_f} + C_2 \left( \frac{Y}{Y_f} \right)^2 \right), \quad (22)$$

where  $Y/Y_f$  is the ratio of the top quark Yukawa coupling squared to its fixed point value, and  $C_1$  and  $C_2$  are coefficients that depend on the value of the strong gauge coupling

constant and, for  $\alpha_3(M_Z) \simeq 0.12$ , are approximately given by  $C_1 \simeq 6$  and  $C_2 \simeq 3$ . Hence, the condition  $m_2^2 \simeq -M_Z^2/2$ , together with the large values of  $M_{1/2}$  necessary to achieve unification of couplings, imply a strong correlation between  $\mu$  and  $M_{1/2}$ . This correlation, which is shown in Fig. 2, has profound implications for the sparticle spectrum and the determination of the bottom mass corrections.

In the following, we shall summarize the main features of the Higgs and supersymmetric spectrum. For a more detailed discussion, we refer the reader to [26].

Fig. 2. Supersymmetric mass parameter  $\mu$  as a function of the gaugino mass  $M_{1/2}$  for two different values of the top quark mass in the framework of bottom–top Yukawa unification. Only the lower top quark mass value  $M_t \simeq 150$  GeV leads to an acceptable value of the physical bottom mass, once the supersymmetry breaking-induced bottom mass corrections are included.

The main features of the sparticle spectrum are governed by the three properties explained above: a) Strong  $\mu$ – $M_{1/2}$  correlation, b) Large values of  $M_{1/2} \geq 300$  GeV, and c)  $M_{1/2} \geq m_0$ . This yields:

1) Small mixing in the chargino and neutralino sectors, which naturally follows from properties a) and b). The lightest supersymmetric particle is mainly a bino, with mass  $M_{\tilde{B}} \simeq 0.4M_{1/2}$ . The second–lightest neutralino and the lightest chargino are winos and hence almost degenerate in mass  $M_{\tilde{W}^+} \simeq M_{\tilde{W}^0} \simeq 2M_{\tilde{B}}$ . The heaviest neutralino and

chargino are Dirac (pseudo-Dirac in the neutralino case) particles, with masses approximately equal to the parameter  $|\mu|$ .

2) Strong stop (sbottom)-gluino running mass correlations, with  $M_{\tilde{t}} \simeq M_{\tilde{b}} \simeq 0.75\text{--}0.8M_{\tilde{g}}$  ( $0.85\text{--}0.9M_{\tilde{g}}$ ) for the lightest (heaviest) squark mass eigenstate. The running gluino mass is related to the common gaugino mass by  $M_{\tilde{g}} \simeq 2.6\text{--}2.8 M_{1/2}$ .

3) Large mixing in the stau sector, due to the off-diagonal left-right stau matrix element  $m_{\tilde{\tau}_{LR}}^2 \simeq -h_{\tau}\mu v_2 \simeq -m_t\mu/\sqrt{3}$ . The large stau mixing leads to a lower bound on  $m_0$ ,

$$m_0^2 \geq -0.15M_{1/2}^2 + \left((0.15M_{1/2}^2)^2 + \mu^2 m_t^2/3\right)^{1/2} \quad (23)$$

in order to avoid a stau being the lightest supersymmetric particle.

4) The Higgs spectrum is characterized by relatively low values for the CP-odd Higgs mass,  $m_A \leq M_{\tilde{t}} (\alpha/5)^{1/2}$ , where  $\alpha$  is the coefficient characterizing the dependence of  $m_{H_1}^2 - m_{H_2}^2$  on  $M_{1/2}^2$ , Eq. (16). Observe that, due to the characteristic values of  $\alpha \simeq 0.2$  (0.1) for  $M_t \simeq 190$  (160) GeV, if the squark masses are lower than a few TeV, the CP-odd and charged Higgs masses will be lower than a few hundred GeV.

5) The lightest CP-even scalar would become lighter than the Z-boson if the CP-odd Higgs scalar were light,  $m_A \leq 150$  GeV. However, in the minimal supergravity SO(10) model, such low values of  $m_A$  induce large values of the  $b \rightarrow s\gamma$  decay rate, since the chargino contribution enhances the rate with respect to that of the Standard Model with an extra Higgs doublet, if the correct bottom mass predictions are required. Therefore, in the minimal supergravity SO(10) model the lightest CP-even Higgs mass is larger than the Z boson mass. We shall come back to this issue in section 5.

## 5 Bottom Mass Predictions and the $b \rightarrow s\gamma$ Decay Rate

In section 3 we have mentioned the possibility of inducing non-negligible one-loop corrections to the running bottom mass, Eq. (9). In this section we want to explicitly show which are the relevant one-loop contributions to the running bottom mass and how they are correlated with the chargino contributions to the  $b \rightarrow s\gamma$  decay rate. As we show in section 3, in order to estimate the size of the bottom mass corrections, we need to compute the supersymmetry breaking-induced effective coupling of the bottom quark to the  $H_2$  field, which amounts to computing the coefficient  $K_1$ , with  $\Delta m_b/\tilde{m}_b = K_1 \tan \beta$ . The contributions to the coefficient  $K_1$  come from the sbottom-gluino and stop-chargino

graph contributions to the bottom quark self energies. It is easy to show that the dominant contributions are given by

$$K_1 = \frac{2\alpha_3}{3\pi} M_{\tilde{g}} \mu I(m_{\tilde{b}_1}^2, m_{\tilde{b}_2}^2, M_{\tilde{g}}^2) + \frac{Y_t}{4\pi} A_t \mu I(m_{\tilde{t}_1}^2, m_{\tilde{t}_2}^2, \mu^2), \quad (24)$$

where  $m_{\tilde{q}_i}^2$ , with  $i = 1, 2$ , are the squark mass eigenstates and the integral factor  $I(a, b, c)$  is given by

$$I(a, b, c) = \frac{ab \log(a/b) + bc \log(b/c) + ac \log(c/a)}{(a-b)(b-c)(a-c)}. \quad (25)$$

The renormalization group equation of the  $A_t$  parameter shows that its low energy values are strongly correlated with the universal gaugino mass. In fact, using the relation  $M_{\tilde{g}} \simeq 2.6\text{--}2.8 M_{1/2}$  it follows [26] that

$$\begin{aligned} A_t &\simeq -\frac{2}{3} M_{\tilde{g}}, & \text{for } Y/Y_t \simeq 1 \\ A_t &\simeq -M_{\tilde{g}}, & \text{for } Y/Y_f \simeq 0.6. \end{aligned} \quad (26)$$

Observe that, due to the minus sign appearing in Eq. (26), there is a partial cancellation between the two different contributions to  $K_1$ , which yields a significant reduction in the bottom mass corrections (typically of the order of 25%). As will be shown below, this partial cancellation, although important, is by far not sufficient to render the bottom mass corrections small.

## 5.1 Conditions for a small $K_1$ and Minimal Supergravity

To get an estimate of the size of the bottom mass corrections, it is important to observe that the integral factors  $I(a, b, c)$  are always of the order of the inverse of the largest mass squared appearing in the integral. Since the coupling constant dependent factors of both contributions to  $K_1$  are of order 0.01, and  $\tan \beta$  is of order 50, to get a small bottom mass correction the following properties should be fulfilled [22]:

$$M_{\tilde{g}} \ll m_{\tilde{q}}, \quad \text{and/or} \quad \mu \ll m_{\tilde{q}}, \quad (27)$$

where  $m_{\tilde{q}}$  represents the heaviest third generation squark mass eigenstates. In addition, the requirement of electroweak symmetry breaking implies that the mass parameter  $B$  should be small in comparison to  $m_A$ , unless  $\mu$  itself is much smaller than  $m_A$ . All these requirements may be satisfied by imposing a softly broken Peccei-Quinn symmetry, which implies the smallness of the mass parameter  $\mu$ , together with an approximate continuous  $R$

symmetry, present in the limit  $B \rightarrow 0$ ,  $M_{\tilde{g}} \rightarrow 0$ ,  $A_t \rightarrow 0$ , whose breaking is characterized by the (assumed) small parameter

$$\epsilon_R \simeq \frac{B}{m_{\tilde{q}}} \simeq \frac{A_t}{m_{\tilde{q}}} \simeq \frac{M_{\tilde{g}}}{m_{\tilde{q}}} . \quad (28)$$

As we shall discuss in section 6, the above conditions may only be reached by relaxing the condition of universality of the soft supersymmetry breaking parameters at the grand unification scale.

In the framework of minimal supergravity, with exact unification of the third generation quark and lepton Yukawa couplings, the strong correlations between the parameters  $\mu$ ,  $A_t$ ,  $M_{\tilde{g}}$  and the third generation squark masses –derived from their renormalization group equations and the condition of a proper breakdown of the electroweak symmetry– imply that the above symmetries are not present in the low energy spectrum. Hence, the bottom mass corrections are large within this framework [26]:

$$\frac{\Delta m_b}{\tilde{m}_b} \simeq 0.0045 \tan \beta \left( \frac{\mu}{M_{1/2}} \right) . \quad (29)$$

Taking into account Eq. (22) and the numerical results from Fig. 1, we see that the corrections are of order 45 % for  $M_t \simeq 190$  GeV and of order 20 % for  $M_t \simeq 150$  GeV.

The corrections are sufficiently large to rule out any solution with  $\tilde{M}_b < M_b$ . This is simply due to the impossibility to accommodate a physical bottom mass in the experimentally allowed range, while keeping the top Yukawa coupling in the perturbative domain at energies of the order of the grand unification scale. Hence, the coefficient  $K_1$  should be negative, implying that the only acceptable branch is that with negative (positive) values of  $M_{\tilde{g}} \times \mu$  ( $A_t \times \mu$ ). Consequently, an upper bound on the top quark mass  $M_t$  may be obtained. This corresponds, for a given value of the strong gauge coupling, to the maximum value of the top quark Yukawa coupling (and  $\tan \beta$ ) consistent with a physical bottom mass  $M_b$  equal to its lower experimental bound  $M_b^L \simeq 4.6$  GeV. There are small uncertainties in the computation of this upper bound, associated with the size of the low energy threshold corrections to the top quark mass, small tau mass corrections analogous to the bottom ones and QCD scale uncertainties. Conservative upper bounds for the top quark mass are given by [26]

$$M_t \leq 165 \text{ (175) (185) GeV,} \quad \text{for } \alpha_3(M_Z) = 0.12 \text{ (0.125) (0.13).} \quad (30)$$

These bounds go rapidly down if the bottom mass is larger than 4.6 GeV. Let us remark again that these bounds do not apply in general in the supersymmetric SO(10) model,



but are only a consequence of the particular supersymmetry breaking scheme under study. Relaxing the high energy boundary conditions for the soft supersymmetry breaking parameters, these bounds on  $M_t$  may be diluted.

## 5.2 $b \rightarrow s\gamma$ Decay Rate

The dominant supersymmetric contributions to the  $b \rightarrow s\gamma$  decay rate have been recently analysed by several authors [29]–[33]. In the Minimal Supersymmetric Standard Model, there is a contribution coming from the charged Higgs, which, for low values of the charged Higgs mass, enhances the Standard Model decay rate. The dominant effect from supersymmetric particles comes from the one-loop chargino–stop contributions to the  $b_R \rightarrow s_L\gamma$  transition. In the supersymmetric limit,  $\tan\beta = 1$  and  $\mu = 0$ , the chargino contributions exactly cancel the  $W^\pm$  and charged Higgs ones, and the  $b \rightarrow s\gamma$  transition element vanishes. This behaviour is not preserved once supersymmetry is broken.

In particular, the large  $\tan\beta$  scenario is far from being close to the supersymmetric limit and the dominant chargino contribution to the  $b \rightarrow s\gamma$  decay rate may have both signs. In general, in the large  $\tan\beta$  regime it is proportional to

$$A_{\gamma,g} \simeq \frac{m_t^2}{m_{\tilde{t}}^2} \frac{A_t \mu}{m_{\tilde{t}}^2} \tan\beta \, g_{\gamma,g} \left( \frac{m_{\tilde{t}}^2}{\mu^2} \right), \quad (31)$$

where  $A_\gamma$  and  $A_g$  are the coefficients of the effective operators for  $bs$ –photon and  $bs$ –gluon interactions, as defined in Ref. [30],  $g_{\gamma,g}(x)$  is a function of  $x$  proportional to the derivative of the function  $f_{\gamma,g}^{(3)}(x)$  defined in Ref. [30], and we have assumed a small mixing in the stop sector, with mass eigenstates  $m_{\tilde{t}_{1(2)}} = m_{\tilde{t}}^2 + (-)A_t m_t$ . From Eq. (31) it follows that the sign of the chargino contribution to the  $b \rightarrow s\gamma$  decay amplitude depends on the sign of  $A_t \times \mu$ , and hence is correlated in sign with the bottom mass corrections discussed above. Observe that the chargino (charged Higgs) contribution to the decay amplitude is always small if the supersymmetric (charged Higgs) spectrum is sufficiently heavy.

One can show that for positive (negative) values of  $A_t \times \mu$  the supersymmetric rate becomes larger (smaller) than that of the Standard Model plus one extra Higgs doublet. Since in the minimal supergravity SO(10) model, once the supersymmetry breaking-induced bottom mass corrections are included, positive values of  $A_t \times \mu$  are required to obtain acceptable values for the physical bottom mass, then, the  $b \rightarrow s\gamma$  decay rate may be significantly enhanced with respect to the SM one. Consequently, one can obtain sparticle mass bounds by requiring the decay rate to be smaller than the present experimental bounds,  $BR(b \rightarrow s\gamma) < 5.4 \times 10^{-4}$  [34]. One should recall, however, that there are

theoretical uncertainties associated with the decay rate computations, which may be as large as 20–30% [35]. Taking them into account at the  $2\text{-}\sigma$  level [26], for a top quark mass  $M_t$  of the order of 150 GeV, the following lower bound on the universal mass parameter  $M_{1/2}$  may be obtained:

$$M_{1/2} \geq 550 \text{ GeV}. \quad (32)$$

This implies a lower bound for the masses of the third generation squarks, charginos and charged Higgses larger than the ones already demanded by the condition of electroweak symmetry breaking,

$$m_{\tilde{q}} \geq 1.2 \text{ TeV}, \quad m_{\tilde{\chi}^+} \geq 450 \text{ GeV}, \quad m_{H^+} \geq 180 \text{ GeV}. \quad (33)$$

Observe that, in this case, the lightest CP-even Higgs mass is constrained to be in the range  $m_h \simeq 110\text{--}130$  GeV.

## 6 Non-Universal Soft Supersymmetry Breaking Parameters

In the above, we have analysed in detail the implications of having universal soft supersymmetry breaking parameters at the grand unification scale. In this case, the bottom mass corrections and the chargino contributions to the  $b \rightarrow s\gamma$  decay rate are correlated in sign and they are sufficiently large to put constraints on the particle spectrum as well as an upper bound on the top quark mass as a function of the strong gauge coupling. The phenomenological implications of relaxing the condition of universality depend on the nature of the soft supersymmetry breaking parameters. Instead of performing a detailed study of the general case, we shall concentrate on understanding which should be the pattern of supersymmetry breaking parameters at the grand unification scale in order to induce only small bottom mass corrections (lower than, say, 10% for a heavy top quark) and small chargino contributions to the  $b \rightarrow s\gamma$  decay rate.

We assume that the universality of the soft supersymmetry breaking gaugino masses is kept and we investigate possible violations of the universality condition in the scalar sector. As was shown in section 5, to obtain an effective cancellation of the bottom quark mass corrections and of the chargino contributions to the  $b \rightarrow s\gamma$  decay rate, we need either small values of  $A_t$  and  $M_{\tilde{g}}$  and/or small values of  $\mu$  in comparison with the squark masses. Hence, the soft supersymmetry breaking parameters should fulfil the following property

$$A_{t,b}(0) \simeq B(0) \simeq M_{1/2} \ll m_S(0), \quad (34)$$

where  $m_S(0)$  represents the characteristic scalar soft supersymmetry breaking mass parameters. Assuming bottom–top–tau Yukawa coupling unification and the fulfilment of Eq. (34), and neglecting the small tau Yukawa coupling effects in the bottom quark Yukawa coupling renormalization group running as well as the small right top–bottom hypercharge difference, the following approximate analytical solutions for the Yukawa couplings and mass parameters are obtained,

$$\begin{aligned} m_{H_2}^2 &= m_{H_2}^2(0) - 3I_U, & m_{H_1}^2 &= m_{H_1}^2(0) - 3I_D - I_L, \\ m_Q^2 &= m_Q^2(0) - I_U - I_D, & m_U^2 &= m_U^2(0) - 2I_U, \\ m_D^2 &= m_D^2(0) - 2I_D, & Y_b \simeq Y_t &= \frac{4\pi Y(0)E}{4\pi + 7Y(0)F}, \end{aligned} \quad (35)$$

where

$$\begin{aligned} I_U &= \frac{Y}{7Y_f} (m_{H_2}^2(0) + m_Q^2(0) + m_U^2(0)) \\ &+ \frac{1}{10} (m_{H_1}^2(0) - m_{H_2}^2(0) + m_D^2(0) - m_U^2(0)) \left[ \frac{5Y}{7Y_f} + \left(1 - \frac{Y}{Y_f}\right)^{5/7} - 1 \right], \\ I_D &= \frac{Y}{7Y_f} (m_{H_1}^2(0) + m_Q^2(0) + m_D^2(0)) \\ &- \frac{1}{10} (m_{H_1}^2(0) - m_{H_2}^2(0) + m_D^2(0) - m_U^2(0)) \left[ \frac{5Y}{7Y_f} + \left(1 - \frac{Y}{Y_f}\right)^{5/7} - 1 \right], \\ I_L &\simeq \frac{1}{6} (m_{H_1}^2(0) + m_L^2(0) + m_E^2(0)) \left[ 1 - \left(1 - \frac{Y}{Y_f}\right)^{1/4} \right]. \end{aligned} \quad (36)$$

In the above,  $Y = Y_t \simeq Y_b$ , with  $Y_i = h_i^2/4\pi$ ,  $E$  and  $F$  are functions of the gauge couplings –which at the weak scale take values  $E \simeq 14$  and  $F \simeq 290$ – and  $Y_f$  is the fixed point value for the top quark Yukawa coupling,  $Y_f \simeq 4\pi E/7F$ , obtained whenever  $Y(0) \geq 0.1$  and whose associated mass expression at the two–loop level is given in Eq. (12). The functions  $I_U$  and  $I_D$  are obtained from an exact solution to the coupled renormalization group equations in the limit of negligible tau Yukawa coupling effects. The function  $I_L$  provides a good approximation to the Yukawa coupling effect in the range  $Y/Y_f \simeq 0.6$ – $0.95$ , and has been obtained assuming that  $Y_\tau$  rapidly acquires its low energy hierarchy with respect to  $Y_b$  and  $Y_t$ .

The interrelation among the different Higgs, squark and slepton masses is quite relevant. For instance, even if we start with values of  $m_{H_2}^2(0)$  lower than  $m_{H_1}^2(0)$  at the grand unification scale, depending on the relation between the squark mass parameters  $m_U^2(0)$

and  $m_D^2(0)$ , one may or may not find an acceptable solution. Close to the infrared fixed point of the top quark mass, Eq. (12), the following relations

$$\begin{aligned} m_Q^2 + m_U^2 + m_{H_2}^2 &\simeq 0 \\ m_Q^2 + m_D^2 + m_{H_1}^2 &\simeq 0 \end{aligned} \quad (37)$$

are fulfilled, implying a strong correlation between the low energy values of the Higgs and squark mass parameters. In particular, for large values of the squark masses  $m_{\tilde{q}}^2 \gg M_Z^2$ , since  $m_2^2 \simeq -M_Z^2/2$ , the above yields:

$$\mu^2 \simeq -m_{H_2}^2 \simeq m_Q^2 + m_U^2. \quad (38)$$

Hence, for values of the top quark mass close to its fixed point value, the bottom mass corrections and the chargino contributions to the  $b \rightarrow s\gamma$  decay rate will only be suppressed if  $M_{\tilde{g}}$  is much smaller than the top and bottom squark masses. Since the gluino mass should be larger than 140 GeV, then, to have bottom mass corrections smaller than 10%, the heavy squark mass values should be larger than 1–2 TeV (observe that, since we are assuming universality of the gaugino masses, the gluino mass constraint may be directly inferred from the chargino mass constraints, without relying on the CDF bounds). Therefore, the top quark mass may be close to its fixed point value, but a very heavy squark spectrum is needed. Observe that this is a general statement based only on the assumption of dominance of the scalar soft supersymmetry breaking terms and has no dependence on the supersymmetry breaking scheme. Larger gaugino masses do not help, since they not only increase the bottom mass corrections, but they also make  $|m_{H_2}^2|$  larger.

Away from the infrared fixed point, the situation becomes more relaxed. In general, the closer one gets to the fixed point value, the heavier should the spectrum be. In order to minimize the bottom mass corrections with low values of the squark masses, the following condition needs to be fulfilled:

$$m_{H_2}^2(0) \left( \frac{7}{3} \frac{Y_f}{Y} - 1 \right) \simeq m_Q^2(0) + m_U^2(0) - \frac{7}{10} \frac{Y_f}{Y} \left( 1 - \frac{5Y}{7Y_f} - \left( 1 - \frac{Y}{Y_f} \right)^{5/7} \right) \Delta_{DU}, \quad (39)$$

where  $\Delta_{DU} = m_D^2(0) - m_U^2(0) + m_{H_1}^2(0) - m_{H_2}^2(0)$ . The above condition ensures that  $\mu^2 \simeq m_{H_2}^2 \ll m_{\tilde{q}}^2$ . In addition, the condition  $m_{H_1}^2 > m_{H_2}^2$  implies

$$m_{H_1}^2(0) - m_{H_2}^2(0) > \frac{1}{6} M_{E_{eff}}^2(0) \left[ 1 - \left( 1 - \frac{Y}{Y_f} \right)^{1/4} \right] + \frac{3}{5} \left[ 1 - \left( 1 - \frac{Y}{Y_f} \right)^{5/7} \right] \Delta_{DU}. \quad (40)$$

Finally, in order to avoid tachyonic solutions in the squark sector, we need,

$$\begin{aligned}
m_U^2(0) &> \frac{2}{3}m_{H_2}^2(0), \\
m_Q^2(0) &> \frac{2}{3}m_{H_2}^2(0) + \frac{1}{5}\Delta_{DU} \left(1 - \left(1 - \frac{Y}{Y_f}\right)^{5/7}\right) \\
m_D^2(0) &> \frac{2}{3}m_{H_2}^2(0) + \frac{2}{5}\Delta_{DU} \left(1 - \left(1 - \frac{Y}{Y_f}\right)^{5/7}\right).
\end{aligned} \tag{41}$$

The requirement that  $\mu^2$  is not correlated with the squark masses, Eq. (39), implies, for a given top quark mass, a very precise relationship between the different supersymmetry breaking parameters at the grand unification scale, which may only be fulfilled in particular supersymmetry breaking scenarios. The necessity of fulfilling the additional constraints given above reduces even more the degree of arbitrariness of the supersymmetry breaking parameters at high energies. The further away from the top quark mass fixed point we are, the easier the conditions are to fulfil. For example, for a top quark mass  $M_t \simeq 175$  GeV, which corresponds to  $Y/Y_f \simeq 0.8$ , the above conditions may be fulfilled by a particularly simple set of supersymmetry breaking parameters: All the scalars acquire a universal soft supersymmetry breaking term  $m_0^2$ , apart from  $m_{H_1}^2(0) \simeq 2m_0^2$ .

One may wonder about the source of the non-universality of the soft supersymmetry breaking scalar masses. In principle, the SO(10) symmetry assures that all squark and sleptons belonging to a single representation of SO(10) acquire a common soft supersymmetry breaking term  $m_0^2$ , while the two Higgs doublets, belonging to a 10 of SO(10), have a common supersymmetry breaking mass  $m_H^2(0)$  at the grand unification scale. There may be, however, additional sources of supersymmetry breaking at the SO(10) breaking scale. A particular natural one is the presence of a  $D$ -term associated with the necessary  $U(1)_X$  gauge symmetry breakdown to reduce the rank of the SO(10) group to that of SU(5). If this term were present, it would break supersymmetry in a very specific way. The following boundary conditions would be obtained in this case [36],

$$\begin{aligned}
m_{H_1}^2(0) &= m_H^2(0) + 2m_X^2 & m_{H_2}^2(0) &= m_H^2(0) - 2m_X^2 \\
m_U^2(0) &= m_Q^2(0) = m_E^2(0) = m_0^2 + m_X^2 & m_D^2(0) &= m_L^2(0) = m_0^2 - 3m_X^2,
\end{aligned} \tag{42}$$

where  $m_X$  is the extra supersymmetry breaking term associated with the SO(10) gauge symmetry breaking. Observe that scalar fields transforming with the same quantum numbers under  $SU(5)$  have degenerate masses. In this specific case,  $\Delta_{DU} = 0$  and the

above conditions take a particularly simple form. In particular, Eq. (39) leads to

$$2m_0^2 \simeq m_H^2(0) \left( \frac{7Y_f}{3Y} - 1 \right) - \frac{14Y_f}{3Y} m_X^2. \quad (43)$$

In addition, the requirement  $m_{H_1}^2 > m_{H_2}^2$  puts a lower bound on  $m_X^2$ , while the requirement of a positive  $m_D^2$  puts an upper bound on  $m_X^2$ ; these are given by

$$\frac{72Y/Y_f}{7 \left[ 1 - (1 - Y/Y_f)^{1/4} \right]} + 2 > \frac{m_H^2(0)}{m_X^2} > \frac{2(1 + 5Y/7Y_f)}{(1 - Y/Y_f)}. \quad (44)$$

The above conditions cannot be fulfilled for  $Y/Y_f > 0.88$  ( $M_t \geq 185$ – $190$  GeV for  $\alpha_3(M_Z) \simeq 0.12$ – $0.13$ ), while for  $Y/Y_f \simeq 0.8$  ( $M_t \simeq 175$ – $180$  GeV) they are only fulfilled for  $m_H^2(0) \simeq 20 m_X^2$  ( $m_0^2 \simeq 0.8 m_H^2(0)$ ).

## 7 Conclusions

In this talk, we have analysed the radiative breakdown of the electroweak symmetry within the Minimal Supersymmetric Standard Model, in the case in which the three Yukawa couplings of the third generation unify at the grand unification scale. We have shown that, due to large bottom quark mass corrections induced through the supersymmetry breaking sector of the theory at the one-loop level, the top quark mass predictions depend strongly on the nature of the soft supersymmetry breaking parameters at low energies. Moreover, the chargino contribution to the bottom mass corrections is strongly correlated with the contribution of these sparticles to the  $b \rightarrow s\gamma$  decay rate.

In the interesting case of universal soft supersymmetry breaking parameters at the grand unification scale, the requirement of electroweak symmetry breaking leads to strong correlations between the mass parameter  $\mu$  and the gaugino masses. The universal scalar mass is constrained to be smaller than the universal gaugino mass, with  $M_{1/2}$  larger than 300 GeV. The result is a heavy sparticle spectrum with strong correlations between the different mass parameters. These correlations allow a precise computation of the bottom mass corrections, which turn out to be very significant. In fact, an upper bound on the top quark mass is derived,  $M_t \leq 165$  (175) (185) GeV for  $\alpha_3(M_Z) \simeq 0.12$  (0.125) (0.13). Moreover, in order to have the  $b \rightarrow s\gamma$  decay rate within its experimental bounds, the common gaugino mass needs to be further constrained,  $M_{1/2} > 550$  GeV. This implies a very heavy squark and gaugino spectrum, with the lightest squark mass larger than 1 TeV. A relatively light Higgs spectrum is obtained, with the lightest CP-even Higgs mass in the range 110–130 GeV and a charged Higgs mass bounded to be  $C_{\tilde{q}} m_{\tilde{q}} \geq m_{H^\pm} \geq 180$

GeV, where  $C_{\tilde{q}} = 0.15\text{--}0.2$  and  $m_{\tilde{q}}$  is the characteristic third generation squark masses. For squark masses lower than 2 TeV, for example, the charged Higgs mass cannot be larger than 400 GeV.

The above conditions may be modified if non-universal soft supersymmetry breaking parameters at the grand unification scale are considered. We have shown that it is possible to minimize the supersymmetric bottom mass corrections and the contributions to the  $b \rightarrow s\gamma$  decay rate, which allows to accommodate a heavier top quark mass. This demands the presence of light gauginos in the spectrum. The squarks and sleptons are in general heavy, with masses larger than or of the order of 1 TeV. A very heavy squark and slepton spectrum may only be avoided if very specific relations between the soft supersymmetry breaking parameters are fulfilled. Nevertheless, these relations may only be satisfied if there is a departure of the top quark mass from its infrared fixed point value. For example, if the only source of non-universality is associated with the breakdown of the SO(10) symmetry down to SU(5) via the corresponding U(1)  $D$ -term, the necessary requirements to avoid a heavy squark spectrum may be fulfilled only if  $M_t \leq 185\text{--}190$  GeV for  $\alpha_3(M_Z) \simeq 0.12\text{--}0.13$ , and for a very specific relation among the high energy mass parameters,  $m_0^2 \simeq 0.8m_H^2(0) \simeq 16m_X^2$ , where  $m_0^2$  and  $m_H^2(0)$  are the universal squark and Higgs mass parameters before the SO(10) breakdown, while  $m_X^2$  is the  $D$ -term contribution. In the general case the condition of bottom-top Yukawa coupling unification requires specific relations among the soft supersymmetry breaking parameters of the theory, which demand a departure from the infrared fixed point solution of the top quark mass in order to allow moderate values for the squark mass spectrum.

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